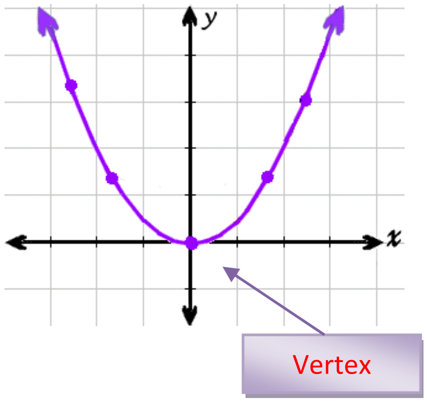
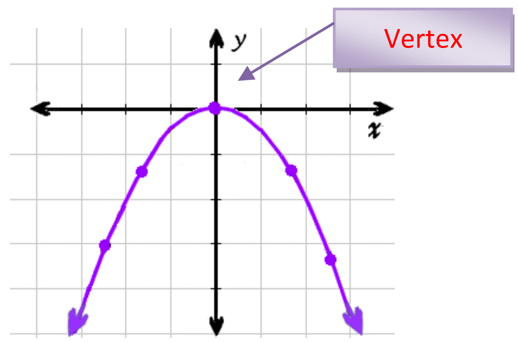
Vertex Form of a Quadratic Function

$$y - k = a(x - h)^2$$
Vertex: (h, k)

When you graph quadratic functions (which we will do later in this unit), you get parabolas that open up or open down.





The **vertex** is either the highest point or the lowest point (depending on the shape) of a quadratic function. When we write the quadratic function in **vertex form**, it is easy to see the coordinates of the **vertex**.

Example 1

Write the quadratic function $y = x^2 - 6x + 11$ in **vertex form** and identify the **vertex**.

Step 1- Move the number (the c in $y = ax^2 + bx + c$) over to the side with y. Leave some space before the $=$ sign.

$$y = x^2 - 6x + 11$$
$$y - 11 \qquad = x^2 - 6x$$

Step 2- We need a (the coefficient of x^2) to be 1. If it isn't, then factor out whatever a is from the right side (don't worry about the left side).

We don't have to worry about this because $a = 1$ for us. We can go on to Step 3.

Step 3- Begin to complete the square on the right side.

$$y - 11 + \square = x^2 - 6x + \square$$

$$(-6) \div 2 = -3 \longrightarrow (-3)^2 = 9$$

$$y - 11 + \square = x^2 - 6x + 9$$

Step 4- Add to the left side to balance what you added to the right side in Step 3. This won't be tricky this time (it will be in the next example).

$$y - 11 + \square = x^2 - 6x + 9$$
$$y - 11 + 9 = x^2 - 6x + 9$$

We added 9 to the right side, so we will also add 9 to the left side

Step 5- Finish completing the square on the right side and the result will be in **vertex form**.

$$y - 11 + 9 = x^2 - 6x + 9$$

$$\boldsymbol{y - 2 = (x - 3)^2}$$

Step 6- Use $y - k = a(x - h)^2$ to identify the **vertex** (h, k).

$$y - 2 = (x - 3)^2$$

The vertex is $(3, 2)$

Example 2

Write the quadratic function $y = 3x^2 - 12x + 1$ in **vertex form** and identify the **vertex**.

Step 1- Move the number (the c in $y = ax^2 + bx + c$) over to the side with y. Leave some space before the $=$ sign.

$$y = 3x^2 - 12x + 1$$
$$y - 1 \qquad = 3x^2 - 12x$$

Step 2- We need a (the coefficient of x^2) to be 1. If it isn't, then factor out whatever a is from the right side (don't worry about the left side).

Right now, $a = 3$. We will factor out 3 from the right side and leave some space for Step 3.

$$y - 1 \qquad = 3x^2 - 12x$$
$$y - 1 \qquad = 3(x^2 - 4x \qquad)$$

Step 3- Begin to complete the square on the right side.

$$y - 1 + \square \quad = 3(x^2 - 4x + \square)$$

$$(-4) \div 2 = -2 \longrightarrow (-2)^2 = 4$$

$$y - 1 + \square \quad = 3(x^2 - 4x + 4)$$

Step 4- Add to the left side to balance what you added to the right side in Step 3.

Be careful! We filled in with a 4, but we actually added a total of 12 to the right side (remember the 3 we factored out in Step 2?).

$$y - 1 + \square \quad = 3(x^2 - 4x + 4)$$

$$y - 1 + 12 \quad = 3(x^2 - 4x + 4)$$

We added 3 • (4) = 12 to the right side, so we will add 12 to the left side

Step 5- Finish completing the square on the right side and the result will be in **vertex form**.

$$y + 11 = 3(x^2 - 4x + 4)$$

$$\boldsymbol{y + 11 = 3(x - 2)^2}$$

Step 6- Use $y - k = a(x - h)^2$ to identify the **vertex** (h, k).

$$y + 11 = 3(x - 2)^2$$
$$y - (-11) = 3(x - 2)^2$$

The vertex is $(2, -11)$